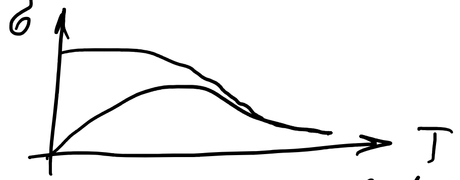
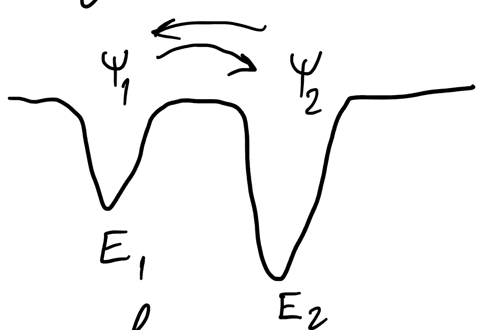
Ac conductivity at strong disorder

In an insulator the conductivity vanishes at $T=0$. However, the ac conductivity may remain finite.



Deeply in the insulating state it comes from pairs of closely localised states with distance between each other much smaller than average.

Consider one such pair.



(For example, a pair of impurities)

$$\Psi_{\pm} = C_1 \Psi_1 + C_2 \Psi_2$$

$$\begin{pmatrix} E_1 & I \\ I^* & E_2 \end{pmatrix} \Psi = E \Psi$$

$$E_{\pm} = \frac{E_1 + E_2}{2} \pm \left[\left(\frac{E_1 - E_2}{2} \right)^2 + |I|^2 \right]^{\frac{1}{2}}$$

$$\frac{C_1}{C_2} = \frac{2I}{E_1 - E_2 \pm \left[(E_1 - E_2)^2 + 4|I|^2 \right]^{\frac{1}{2}}}$$

If $|E_1 - E_2| \gg I$, then wells stay pretty much isolated. In the opposite limit,

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much isolated. In the opposite limit,

$$\Psi_{\pm} = \frac{\Psi_1 \pm \Psi_2}{\sqrt{2}}$$

Let's apply an external electromagnetic field

$$\vec{E} = \vec{E}_0 \cos(\omega t) = \frac{1}{2} \vec{E}_0 \left(e^{i\omega t} + e^{-i\omega t}\right)$$

The perturbation Hamiltonian

$$\hat{V} = -\vec{E}\vec{d} = -e\vec{E}\vec{r}$$

The rate of the ac-field-induced transitions between states $|+\rangle$ and $|-\rangle$, i.e. from state $|-\rangle$ to state $|+\rangle$, assuming that the former is occupied and the latter is not.

$$\Gamma_{-\to+} = \frac{2\pi}{\hbar} \frac{1}{4} |e E_0 \langle -|\vec{r}|+\rangle|^2 \delta(\hbar\omega - (E_+ - E_-))$$

When an electron transitions, it absorbs a photon with energy $\hbar\omega$.

The energy absorbed per time

$$q = \frac{2\pi}{\hbar} \frac{\hbar\omega}{4} |e E_0 \langle -|\vec{r}|+\rangle|^2 \delta(\hbar\omega - (E_+ - E_-)) (n_- - n_+)$$

Occupation numbers for states $|-\rangle$ and $|+\rangle$

There are 3 possibilities
1. The pair is empty = no electrons

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   Total energy $= 0$

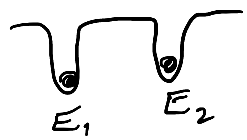
2. 1 state is occupied, and the other is empty

   The total energy is either E^+ or E^-

3. Both states are occupied

   $$\tilde{E} = E_1 + E_2 + \frac{e^2}{\varkappa r}$$

   Coulomb interaction



The partition function

$$Z = 1 + e^{-\frac{E^+-\mu}{T}} + e^{-\frac{E^--\mu}{T}} + e^{-\frac{\tilde{E}-2\mu}{T}}$$

Then $n_- = \frac{1}{Z} e^{-\frac{E^--\mu}{T}}$, $n_+ = \frac{1}{Z} e^{-\frac{E^+-\mu}{T}}$

$$q = \frac{2\pi}{\hbar} \frac{\hbar\omega}{4Z} \left| eE_0 \langle - | \vec{r} | + \rangle \right|^2 \delta\left(\hbar\omega - (E^+ - E^-)\right) e^{-\frac{E^--\mu}{T}} \left(1 - e^{-\frac{\hbar\omega}{T}}\right)$$

Let's assume $T \ll \hbar\omega$

Then $n_- - n_+ \approx 1$

Assume $E^- - \mu < 0$, to ensure that state $|-\rangle$ is filled.

Also, we assume that $E^- - \mu < \tilde{E} - 2\mu$

The conductivity is given by

$$\sigma(\omega) = \frac{1}{E_0^2} \int d\vec{r} \int dE_1\, dE_2\, g(E_1)\, g(E_2)\, q(E_1, E_2, \vec{r})$$

. the energy

$$\sigma(\omega) = \frac{1}{E_0^2}\int d\vec{r}\int dE_1\, dE_2\, \underbrace{g(E_1)}_{=\, g_0}\, \underbrace{g(E_2)}_{=\, g_0} \ldots$$

in the energy interval of interest

Change variables:

$$E^- = \frac{E_1+E_2}{2} - \frac{1}{2}\left[(E_1-E_2)^2 + 4I^2\right]^{\frac{1}{2}}$$

$$W = E^+ - E^- = \left[(E_1-E_2)^2 + 4I^2\right]^{\frac{1}{2}}$$

The Jacobian of the variable transformation:

$$\begin{vmatrix} \frac{\partial E^-}{\partial E_1} & \frac{\partial E^-}{\partial E_2} \\ \frac{\partial W}{\partial E_1} & \frac{\partial W}{\partial E_2} \end{vmatrix} = \begin{vmatrix} \frac{1}{2} - \frac{1}{2}\frac{\Delta}{\sqrt{\Delta^2+4I^2}} & \frac{1}{2} + \frac{1}{2}\frac{\Delta}{\sqrt{\Delta^2+4I^2}} \\ \frac{\Delta}{\sqrt{\Delta^2+4I^2}} & -\frac{\Delta}{\sqrt{\Delta^2+4I^2}} \end{vmatrix}$$

where $\Delta = E_1 - E_2$

$$= -\frac{\Delta}{\sqrt{\Delta^2+4I^2}} = \frac{-\sqrt{W^2-4I^2}}{W}$$

$$dE_1\, dE_2 \to dE^-\, dW \left|\frac{\partial(E_1, E_2)}{\partial(E^-, W)}\right| = dE^-\, dW\, \frac{W}{\sqrt{W^2-4I^2}}$$

What are the integration limits wrt W and E^-?

$$\begin{cases} E^- - \mu < 0 & (1) \\ E^- - \mu < \underbrace{\tilde{E}}_{=\, E_1+E_2+\frac{e^2}{\varkappa r} \,=\, 2E^- + W + \frac{e^2}{\varkappa r}} - 2\mu & (2) \end{cases}$$

$$(2) \to E^- - \mu > -W - \frac{e^2}{\varkappa r}$$

$$-W - \frac{e^2}{\varkappa r} < E^- - \mu < 0$$

E^- integration interval

$$|\langle -|r|+\rangle| = \frac{I}{W} \text{ - Dipole matrix element}$$

$$\sigma(\omega) = 4\pi\int r^2 dr \cdot \int dE^-\, dW\, \frac{W}{\sqrt{W^2-4I^2}}\, \frac{2\pi}{\hbar}\, \frac{\hbar\omega}{4}\, e^2 E_0^2\, \frac{I^2}{W^2}\, \frac{r^2}{3}\, \delta(\hbar\omega - W)\, \frac{e^{-\frac{E^- - \mu}{T}}}{Z}$$

$$\sigma(\omega) = 4\pi \int r^2 dr \cdot \int dE^- \, dW \frac{W}{\sqrt{W^2 - 4I^2}} \frac{2\pi}{\hbar} \frac{\pi\omega}{4} e^2 E_0^2 \frac{\cdots}{W^2} \frac{\cdots}{3} \cdots \quad Z$$

Angle averaging

$$\int dE^- \ldots \to W + \frac{e^2}{\varkappa r}$$

$$\int dW \, \delta(\hbar\omega - W) = 1$$

$$\sigma(\omega) = \frac{2\pi^2}{3} g_0^2 e^2 E_0^2 \hbar^{-1} \int_{r_\omega}^{\infty} dr \cdot r^4 \left(\hbar\omega + \frac{e^2}{\varkappa r}\right) \frac{I_r^2}{\sqrt{(\hbar\omega)^2 - 4I_r^2}}$$

where $r_\omega = \xi \ln \frac{2I_0}{\hbar\omega}$, assuming $I_r = I_0 e^{-\frac{r}{\xi}}$

$$\sigma(\omega) = \frac{2\pi^2}{3} e^2 g_0^2 \xi \omega r_\omega^4 \left(\hbar\omega + \frac{e^2}{\varkappa r_\omega}\right)$$

Neglecting Coulomb interactions,

$$\boxed{\sigma(\omega) \propto \omega^2}$$